Relations and Functions

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q1. Assertion (A): The relation R in a set

$A = \{1, 2, 3, 4\}$ defined by $R = \{(x, y) : 3x - y = 0\}$

have the Domain = {1, 2, 3, 4}

and Range = {3, 6, 9, 12}.

Reason (R): Domain and range of the relation (R) is respectively the set of all first and second entries of the distinct ordered pair of the relation.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q2. Assertion (A): If R is a relation defined on the set of natural numbers N such that $R = \{(x, y): x, y \in N \text{ and } 2x + y = 24\}$, then R is an equivalence relation.

Reason (R): A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.

Answer : (d) Assertion (A) is false but Reason (R) is true.

Q3. Assertion (A): If the relation R defined in A = {1,2,3} by aRb, if $|a^2 - b^2| \leq 5$, then $R^{-1} = R$.

Reason (R): For above relation, domain of R^{-1} = Range of R.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Q4. Assertion (A): A function y = ƒ(x) defined by x^2 - $\cot^{-1}$ y = л, then domain of ƒ(x) = R.

Reason (R): $\cot^{-1} y \in (0, \pi)$.

Answer : (d) Assertion (A) is false but Reason (R) is true.

Q5.

Assertion (A): A function $f : R \to R$ satisfies the equation $f(x) - f(y) = x - y \ \forall\ x, y \in R$ and $f(3) = 2$, then $f(xy) = xy - 1$.

Reason (R): $f(x) = f\left(\frac{1}{x}\right) \forall\ x \in R, x \neq 0$

and $f(2) = \frac{7}{3}$ if $f(x) = \frac{x^2 + x + 1}{x^2 - x + 1}$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Q6.

Assertion (A): The relation R on the set $N \times N$, defined by $(a, b)\, R\, (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation.

Reason (R): Any relation R is an equivalence relation, if it is reflexive, symmetric and transitive.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q7. Assertion (A): The relation R given by R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} on a set A = {1, 2, 3, 4} is not symmetric.

Reason (R): For symmetric relation, $R = R^{-1}$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q8. Assertion (A): The relation ƒ : {1,2,3, 4} → {x,y,z,p} defined by ƒ = {(1, x), (2, y), (3,z)} is a bijective function.

Reason (R): The function ƒ : {1,2,3} → {x,y,z,p} such that ƒ = {(1, x), (2, y), (3, z)} is one-one. **(CBSE SQP 2023-24)**

Answer : (d) Assertion (A) is false but Reason (R) is true.

Assertion (A) The relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as $R = \{(x, y)$: y is divisible by $x\}$ is not an equivalence relation.

Reason (R) The relation R will be an equivalence relation, if it is reflexive, symmetric and transitive.

Assertion (A) If R is the relation defined in set $\{1,2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$, then R is reflexive

Reason (R) The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric.

Let R be any relation in the set A of human beings in a town at a particular time.

Assertion (A) If $R = \{(x, y) : x$ is wife of $y\}$, then R is reflexive.

Reason (R) If $R = \{(x, y) : x$ is father of $y\}$, then R is neither reflexive nor symmetric nor transitive.

Assertion (A) If $A = \{x \in z : 0 \leq x \leq 12\}$ and R is the relation in A given by $R = \{(a, b) : a = b$. Then, the set of all elements related to 1 is $\{1, 2\}$.

Reason (R) If R_1 and R_2 are equivalence relation in a set A, then $R_1 \cap R_2$ is an equivalence relation.

If R is the relation in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even$\}$,

Assertion (A) R is an equivalence relation.

Reason (R) All elements of $\{1, 3, 5\}$ are related to all elements of $\{2, 4\}$.

Assertion (A) The function $f : R^* \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R^* is the set of all non-zero real numbers.

Reason (R) The function $g : N \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto.

Assertion (A) Let A and B be sets. Then, the function $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is bijective.

Reason (R) A function f is said to be bijective, if it is both one-one and onto.

Assertion (A) The modulus function $f : R \to R$ given by $f(x) = |x|$ is neither one-one nor onto.

Reason (R) The signum function $f : R \to R$ given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is bijective.

Assertion (A) Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 are ± 4.

Reason (R) A function $f : A \to B$ is called a one-one function, if distinct elements of A have distinct images in B.

Assertion (A) The function $f : R \to R$ given by $f(x) = x^3$ is injective.

Reason (R) The function $f : X \to Y$ is injective, if $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in X$.

Assertion (A) The function $f(x) = x^2 + bx + c$, where b and c are real constants, describes onto mapping.

Reason (R) Let $A = \{1, 2, 3, \ldots, n\}$ and $B = \{a, b\}$. Then, the number of surjections from A into B is $2^n - 2$.

Assertion (A) Let a relation R defined from set $A = \{1, 2, 5, 6\}$ to A is $R = \{(1, 1), (1, 6), (6, 1)\}$, then R is symmetric relation.

Reason (R) A relation R in set A is called symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in A$.

Assertion (A) The $f : R \to R$ given by $f(x) = [x] + x$ is one-one onto.

Reason (R) A function is said to be one-one and onto, if each element has unique image and range of $f(x)$ is equal to codomain of $f(x)$.

Assertion (A) Let $A = \{2, 4, 6\}$ and $B = \{3, 5, 7, 9\}$ and defined a function $f = \{(2, 3), (4, 5), (6, 7)\}$ from A to B. Then, f is not onto.

Reason (R) A function $f : A \to B$ is said to be onto, if every element of B is the image of some elements of A under f.

Assertion (A) Let a relation R defined from set B to B such that $B = \{1, 2, 3, 4\}$ and $R = \{1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$, then R is transitive.

Reason (R) A relation R in set A is called transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R, \forall\ a, b, c \in A$

ANSWE

9. (a) 10. (d) 11. (d) 12.

Assertion Here, $R = \{(x, y) : y$ is divisible by $x\}$ is a relation in the set $A = \{1, 2, 3, 4, 5, 6\}$.

For reflexive, we know that x is divisible by x, which is true for all $x \in A$.

$\therefore (x, x) \in R$ for all $x \in A$.

So, R is reflexive.

For symmetry, we observe that 6 is divisible by 2 i.e. $(2, 6) \in R$ but 2 is not divisible by 6 i.e. $(6, 2) \notin R$.

So, R is not symmetric.

For transitivity, let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow y$ is divisible by x and z is divisible by y.

$\Rightarrow z$ is divisible by x.

$\Rightarrow (x, z) \in R$

e.g. 2 is divisible by 1 and 4 is divisible by 2.

So, 4 is divisible by 1. So, R is transitive.

Hence, R is not an equivalence relation.

Assertion Let $A = \{1, 2, 3, 4, 5, 6\}$

A relation R is defined on set A is

$$R = \{(a, b) : b = a + 1\}$$

$\therefore \quad R = \{(1, 2,), (2, 3), (3, 4), (4, 5), (5, 6)\}$

Now, $6 \in A$ but $(6, 6) \notin R$

$\therefore$ R is not reflexive.

Reason Given set $A = \{1, 2, 3\}$

A relation R on A is defined as

$$R = \{(1, 2), (2, 1)\}$$

$\because (1, 2) \in R$ and $(2, 1) \in R$.

So, R is symmetric.

Assertion Here R is not reflexive: as x cannot be wife of x.

Reason Here, R is not reflexive; as x cannot be father of x, for any x. R is not symmetric as if x is father of y, then y cannot be father of x. R is not transitive as if x is father of y and y is father of z, then x is grandfather (not father) of z.

Assertion The elements that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is $\{1\}$.

Reason Since, R_1 and R_2 are equivalence relations, therefore $(a, a) \in R_1$, $(a, a) \in R_2$, $\forall\, a \in A$.

This implies that $(a, a) \in R_1 \cap R_2, \forall a$.

Hence, $R_1 \cap R_2$ is reflexive.

Further, $(a, b) \in R_1 \cap R_2 \Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2 \Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$ $\Rightarrow (b, a) \in R_1 \cap R_2$.

Hence, $R_1 \cap R_2$ is symmetric.

Similarly, $(a, b) \in R_1 \cap R_2$ and $(b, c) \in R_1 \cap R_2$ $\Rightarrow (a, c) \in R_1$ and $(a, c) \in R_2 \Rightarrow (a, c) \in R_1 \cap R_2$.

This implies that $R_1 \cap R_2$ is transitive.

Hence, $R_1 \cap R_2$ is an equivalence relation.

Assertion Given that, $A = \{1, 2, 3, 4, 5\}$,

$$R = \{(a, b) : |a - b| \text{ is even}\}$$

Let $a \in A \Rightarrow |a - a| = 0$ (which is even), $\forall\, a$

So, R is reflexive.

Let $(a, b) \in R \;\rightarrow |a - b|$ is even.

$\Rightarrow |a - b| = |-(b - a)| = |b - a|$, therefore $|b - a|$ is also even.

$\Rightarrow (b, a) \in R$. So, R is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow |a - b|$ is even and $|b - c|$ is even.

$\Rightarrow (a - b)$ is even and $(b - c)$ is even.

$\Rightarrow (a - c) = (a - b) + (b - c)$ is even

[$\because$ sum of two even integers is even]

$\Rightarrow |a - c|$ is even $\Rightarrow (a, c) \in R$.

So, R is transitive.

Hence, R is an equivalence relation.

Reason Also, no element of the $\{1, 3, 5\}$ can be related to any element of $\{2, 4\}$, as all elements of $\{1, 3, 5\}$ are odd and all elements of $\{2, 4\}$ are even.

So, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Hence Reason is not correct.

Assertion It is given that $f : R^* \rightarrow R^*$ is defined by

$$f(x) = \frac{1}{x}$$

For one-one, $f(x) = f(y)$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{y}$$

$$\Rightarrow \quad x = y$$

Therefore, f is one-one.

For onto, it is clear that for $y \in R^*$, there exists $x = \frac{1}{y} \in R^*$ (exists as $y \neq 0$) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$$

Therefore, f is onto. Thus, the given function (f) is one-one and onto.

Reason Now, consider function $g : N \rightarrow R^*$ defined by $g(x) = \frac{1}{x}$.

We have, $g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$$\Rightarrow \quad x_1 = x_2$$

Therefore, g is one-one.

Further, it is clear that g is not onto as for $1 \cdot 2 \in R^*$, there does not exist any x in N such that $g(x) = 1 \cdot 2 \Rightarrow \frac{1}{x} = 1 \cdot 2$

$$\Rightarrow \quad x = \frac{1}{1 \cdot 2} \notin N \text{ (domain)}$$

Hence, function g is one-one but not onto.

Assertion Here, $f : A \times B \rightarrow B \times A$ is defined as $f(a, b) = (b, a)$.

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that

$$f(a_1, b_1) = f(a_2, b_2)$$

$$\Rightarrow \quad (b_1, a_1) = (b_2, a_2)$$

$$\Rightarrow \quad b_1 = b_2 \text{ and } a_1 = a_2$$

$$\Rightarrow \quad (a_1, b_1) = (a_2, b_2)$$

Therefore, f is one-one.

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$. [definition of f]

Therefore, f is onto. Hence, f is bijective.

Assertion Here, $f : R \rightarrow R$ is given by

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

It is seen that $f(-1) = |-1| = 1, f(1) = |1| = 1$

Therefore, $f(-1) = f(1)$ but $-1 \neq 1$

Therefore, f is not one-one.

Now, consider $-1 \in R$

It is known that $f(x)=|x|$ is always non-negative.

Thus, there does not exist any element x in domain R such that $f(x)=|x|=-1$.

Therefore, f is not onto.

Hence, the modulus function is neither one-one nor onto.

Reason $f: R \rightarrow R, f(x)=\begin{cases}1, & \text{if} \quad x>0 \\ 0, & \text{if} \quad x=0 \\ -1, & \text{if} \quad x<0\end{cases}$

It is seen that $f(1)=f(2)=1$ but $1 \neq 2$.
Therefore, f is not one-one.

Now, as $f(x)$ takes only three values (1, 0 or -1), therefore for the element -2 in codomain R, there does not exist any x in domain R such that $f(x)=-2$.

Therefore, f is not onto. Hence, the Signum function is neither one-one nor onto.

Assertion Consider $x^2+1=17$

$\Rightarrow \quad x^2=16$

$\Rightarrow \quad x=\pm 4$

Hence, pre-images of 17 are ± 4.

Hence, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Assertion Here, $f: R \rightarrow R$ is given as $f(x)=x^3$.

Suppose $f(x)=f(y)$

where $x, y \in R$

$\Rightarrow \quad x^3=y^3 \quad \ldots(\text{i})$

Now, we try to show that $x=y$.

Suppose $x \neq y$, their cubes will also not be equal.

$$x^3 \neq y^3$$

However, this will be a contradiction to Eq. (i).

Therefore, $x=y$. Hence, f is injective.

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Assertion Given function is

$$f(x)=x^2+bx+c.$$

It is a quadratic equation in x.

So, we will get a parabola either downward or upward.

Hence, it is a many-one mapping and not onto mapping.

Hence, it is neither one-one nor onto mapping.

Reason Total number of functions

$$=(n(B))^{n(A)}=2^n.$$

Clearly, a function will not be onto if all elements of A map to either a or b.

Assertion We have, $A=\{1,2,5,6\}$
and $R=\{(1,1),(1,6),(6,1)\}$
Here, $(1,6) \in R$
$\Rightarrow \quad (6,1) \in R$
Hence, R is symmetric relation.

Assertion Since, greatest integer $[x]$ gives only integer value.

But $f(x)=[x]+x$ gives all real values and there is no repeated value of $f(x)$ for any value of x.

Hence, $f(x)$ is one-one and onto.

Assertion Given that,

$$A=\{2,4,6\},$$
$$B=\{3,5,7,9\}$$

and $\quad R=\{(2,3),(4,5),(6,7)\}$

Here, $f(2)=3$, $f(4)=5$ and $f(6)=7$

It can be seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one but f is not onto as $9 \in B$ does not have a pre-image in A.

Hence, both Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

Assertion We have, $B=\{1,2,3,4\}$
and $R=\{(1,1),(2,2),(3,3),(1,3),(3,1)\}$
Here, $(1,3),(3,1) \in R$
$\Rightarrow \quad (1,1) \in R$
Hence, R is transitive.

Let W be the set of words in the English dictionary. A relation R is defined on W as
$R = \{(x, y) \in W \times W$ such that x and y have at least one letter in common$\}$.

Assertion (A): R is reflexive.

Reason (R): R is symmetric.

Ans. Option (B) is correct.

Explanation: For any word $x \in W$
x and x have atleast one (all) letter in common
$\therefore \quad (x, x) \in R, \forall\, x \in W \therefore R$ is reflexive
Symmetric: Let $(x, y) \in R, x, y \in W$
$\Rightarrow \quad x$ and y have atleast one letter in common
$\Rightarrow \quad y$ and x have atleast one letter in common
$\Rightarrow \quad (y, x) \in R \therefore R$ is symmetric
Hence A is true, R is true; R is not a correct explanation for A.

Let R be the relation in the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } a - b\}$.

Assertion (A): R is a reflexive relation.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in Z$.

Ans. Option (A) is correct.

Explanation: By definition, a relation in Z is said to be reflexive if xRx, $\forall x \in Z$. So R is true.

$a - a = 0 \Rightarrow 2$ divides $a - a \Rightarrow aRa$.

Hence R is reflexive and A is true.

R is the correct explanation for A.

Consider the set $A = \{1, 3, 5\}$.

Assertion (A): The number of reflexive relations on set A is 2^9.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in A$.

Ans. Option (D) is correct.

Explanation: By definition, a relation in A is said to be reflexive if xRx, $\forall x \in A$. So R is true.

The number of reflexive relations on a set containing n elements is $2^{n^2 - n}$.

Here $n = 3$.

The number of reflexive relations on a set $A = 2^{9-3} = 2^6$.

Hence A is false.

Consider the function $f : R \to R$ defined as $f(x) = x^3$

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is a one-one function if co-domain = range.

Ans. Option (C) is correct.

Explanation: $f(x)$ is a one-one function if

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

Hence R is false.

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$\Rightarrow (x_1)^3 = (x_2)^3$

$\Rightarrow x_1 = x_2$

Hence $f(x)$ is one-one.

Hence A is true.

If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B.

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is an onto function.

Ans. Option (C) is correct.

Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f : A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ *i.e.*, $f(1) = 4$, $f(2) = 5$ and $f(3) = 6$.

It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.

So, A is true.

Range of $f = \{4, 5, 6\}$.

Co-domain $= \{4, 5, 6, 7\}$.

Since co-domain $\neq$ range, $f(x)$ is not an onto function. Hence R is false.

Consider the function $f : R \to R$ defined as

$$f(x) = \frac{x}{x^2+1}.$$

Assertion (A): $f(x)$ is not one-one.

Reason (R): $f(x)$ is not onto.

Ans. Option (B) is correct.

Explanation: Given, $f : R \to R$;

$$f(x) = \frac{x}{1+x^2}$$

$$\text{Taking } x_1 = 4,\ x_2 = \frac{1}{4} \in R$$

$$f(x_1) = f(4) = \frac{4}{17}$$

$$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17} \qquad (x_1 \neq x_2)$$

$\therefore$ f is not one-one.

A is true.

Let $y \in R$ (co-domain)

$f(x) = y$

$$\Rightarrow \frac{x}{1+x^2} = y$$

$$\Rightarrow y.(1 + x^2) = x$$

$$\Rightarrow yx^2 + y - x = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

since, $x \in R$,

$$\therefore 1 - 4y^2 \geq 0$$

$$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

So Range $(f) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Range $(f) \neq R$ (Co-domain)

$\therefore f$ is not onto.

R is true.

R is not the correct explanation for A.

Assertion (A) : Let L be the collection of all lines in a plane and R_1 be the relation on L as $R_1 = \{(L_1, L_2) : L_1 \perp L_2\}$ is a symmetric relation.

Reason (R) : A relation R is said to be symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$.

Assertion (A) : Let R be the relation on the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

Reason (R) : A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Assertion (A) : Let $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$, then f is a one-one function.

Reason (R) : A function $g : A \to B$ is said to be onto function if for each $b \in B, \exists\, a \in A$ such that $g(a) = b$.

Assertion (A) : Let function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ be an onto function. Then it must be one-one function.

Reason (R) : A one-one function $g : A \to B$, where A and B are finite set and having same number of elements, then it must be onto and vice-versa.

Answers